DEIM Forum 2014 D5-2

Measuring spike train distance from multichannel spike trains data simulated by coupled escape rate model

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Abstract Estimating the population activity patterns between two or more spike trains is a fundamental problem in studying neural coding in computational neuroscience. In recent years, there are many different methods proposed to build a framework to deal with these problems by using spike train metric. Here we suggest a kernel method for multichannel spike trains that can provide an opportunity to measure spike trains. As kernels can be used for various tasks in machine learning, including regression, clustering and dimension reduction. We believe this method is effective at measuring multichannel spike trains simulated using a distance.

Keyword spike train distance, coupled escape rate model, kernel methods, multichannel spike trains, neuronal coding

1. Introduction

In our brain, there are over 100 billion neurons, and they communicate with each other using pulses by generating characteristic electrical signals called action potentials or spikes. We don't know how these neurons transmit information rapidly and what relationship they have between neural response and stimulus attributes.

We not only want to find the structure of neuron networks, but also want to know information contents of neural signals system. It is therefore important to know how information is coded by neurons. To this end many researchers are working on the analysis of spike trains.

A spike train is a series of spikes or action potentials fired by a neuron. Although there is a method to electrically visualize action potential propagation and network topology in cortical neurons [1], it can't explain how spike works and the temporal structure of spike trains. There is a big motivation to understand how to analyze and decode the information expressed by spike trains in computational neuroscience. Recently, there are some publicly available spike train data. Also there is some work that uses distances to evaluate similarity of spike trains. The aim of this paper is to propose kernels for spike trains. Specifically we use coupled escape rate model (CERM) [2] to simulate spike trains data, and apply the memoryless cross intensity kernel, which is calculated by convolving the spike trains and a smoothing function. We then obtain a distance between spike trains [3][4][5].

The remainder of this paper is organized as follows. The second section describes related work about spike train distances and spike train kernels. In the third and fourth section we will describe the memoryless cross intensity for multichannel spike train model and discuss the coupled escape rate model to simulate multichannel spike trains. And then, a distance is used to evaluate them. Finally, we will give the conclusion.

2.Related work

A way to construct a mathematical framework for spike train is the basic idea to define a distance between spike trains in a spike metric space [6]. Various distances have been proposed for measuring the similarity between spike trains, including the Victor-Purpura distance [6], van Rossum distance [6], ISI (Inter Spike Interval) distance [7] and spike distance [8].

The Victor-Purpura metric [9][10] and the van Rossum metric [9][10] both compare two different responses from the same neurons corresponding to different trials.

3. Model

3.1. Memoryless cross intensity for spike trains

There are many studies in the literature to use inner products to solve general machine learning problems. Here we use two spike train metrics to define inner products on functional representations of spike trains [3][4][5].

Consider two spike trains, x, y, $x=(x_1, x_2, \ldots, x_i)$, $y=(y_1, y_2, \ldots, y_j)$, with $i, j \in N$, and represent the spike train as a sum of Dirac delta functions $x(t) = \sum_i \delta(t - t_i)$, then define the intensity function by convolving the spike train and a smoothing function h, as in $\lambda(t) = \sum_i h(t - t_i)$. Here, t_i is the timing of the ith spike. The inner product can be simply defined as

$$k(x,y) = \int \lambda_x(t)\lambda_y(t)\,dt \tag{1}$$

This is the memoryless cross intensity kernel (mCI kernel) proposed by Antonio Paiva [3]. In our experiments, we used a Gaussian function for h, which is a very common choice in the analysis of spike trains.

3.2. Memoryless cross intensity for multichannel spike trains

Our aim in this paper is to extend the mCI kernel to multichannel spike trains, and evaluate it using simulated data.

Consider a case where there is a pair of spike train metrics x, y. Let $x=(x_1, x_2, \ldots, x_i)$ and $y=(y_1, y_2, \ldots, y_j)$ where each x_i and y_j indicates a spike trains and $i, j \in N$. Here, x_i will be called a component of x.

The most general way to define a kernel is to use the two spike train metrics x and y directly as variables without imposing any structure. As before, a general kernel can be expressed as k (x, y), here we can define the kernel on a pair of their components x_i and y_j as $k(x_i, y_j)$. The memoryless cross intensity for multichannel spike trains can be written as,

$$k(x,y) = \begin{bmatrix} k(x_1,y_1) & k(x_1,y_2) & \cdots & k(x_1,y_N) \\ k(x_2,y_1) & k(x_2,y_2) & \cdots & k(x_2,y_N) \\ \vdots & & \ddots & \vdots \\ k(x_N,y_1) & k(x_N,y_2) & \cdots & k(x_N,y_N) \end{bmatrix} \tag{2}$$

If x = y, we will get a diagonal matrix.

4. Method

It is well known that the firing rate is changed over time in neural code experiments. We use inhomogeneous Poisson process to simulate spike trains. We also use the coupled escape rate model (CERM) [2] to model interactions among neurons. It takes many parameters from a realistic network model.

Here we considered a case in which there are two neurons a, b with synaptic coupling. The coupled escape rate model is defined as follows.

$$\lambda_a(t) = \exp\left[\, u_a + \alpha_a x_a(t) + J_{ab}(t) s_{ab}(t) \,\right] \tag{3}$$

$$\lambda_b(t) = \exp\left[\, u_b + \alpha_b x_b(t) + J_{ba}(t) s_{ba}(t) \,\right] \tag{4}$$

$$\frac{dx_a}{dt} = -\frac{x_a}{\tau_m} + \sum_k \delta(t - t_{a,k}) \tag{5}$$

$$\frac{ds_{ab}}{dt} = -\frac{s_{ab}}{\tau_s} + \sum_k \delta(t - t_{b,k}) \tag{6}$$

$\lambda_i(t)$ is the instantaneous firing rate of neuron i, most parameters are indicated in Figure 1. $t_{i,k}$ is the k-th spike time of neuron i, and $\delta(t)$ is the Dirac delta function. Both the time constants τ_m and τ_s were 10 ms. (5) and (6) were calculated with a time step of 1 ms.

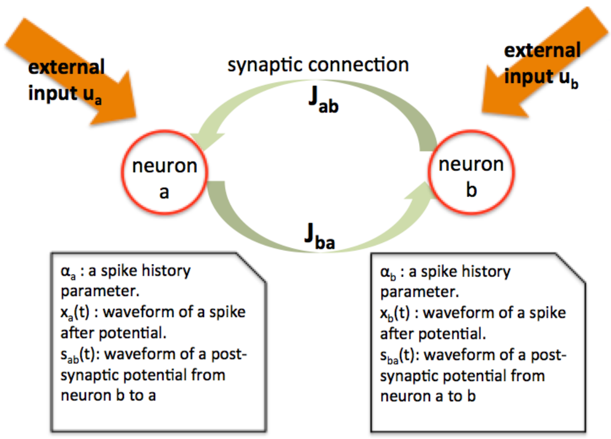

Figure 1 CERM model parameters: $\{u_{a,b},\ a_{a,b},\ J_{ab,ba},\ x_{a,b}(t),\ s_{ab,ba}(t)\}$.

5. Evaluation

In this section, we first use the coupled escape rate model to get the simulation spike trains data. We also compare the average instantaneous firing rate by changing one of parameters in the coupled escape rate model. Specifically, The external input to one neuron is changed. Finally, we show how a distance could be used to evaluate the multichannel mCI in a rigorous manner.

5.1. Simulation data

We used the coupled escape rate model defined in the previous section. There are two neurons used in the simulation that produce Poisson spike trains with time-varying firing rates.

Most of the parameters were summarized in Table 1. The data had been classified into ten conditions by changing u_1=[1.4: 0.2: 3.2]. For each condition, 20 trials were carried out. The time step is set to 1 ms and each trial lasted for 500ms. Our simulation model was run using MATLAB.

Table 1 Summary of parameters used in our simulation model.

a_1, a_2	-0.6, -0.9
J_{12}, J_{21}	-0.5, -0.4
u_1	[1.4 : 0.2 : 3.2]
u_2	1.7
$x_1(1)$, $x_2(1)$	0, 0
$s_{12}(1)$, $s_{21}(1)$	0, 0

With the same stimuli, the neuron will get different respond which one maybe has subtle differences. During each condition, we just change the external input to neuron 1, and get 200 times spike trains data in total. So when u_1= 1.4, we can get 20 times spike trains as t=(t^1, t^2, t^3, … , t^{20}), the same to when u_1= 1.6, another 20 times spike trains as s=(s^1, s^2, s^3, … , s^{20}) and so on.

As the result of the of simulation model, in Figure 1 we compute the average firing rate of multichannel spike trains.

5.2. Evaluation multichannel mCI

Since we get multichannel spike trains t and s using the coupled escape rate model, we should evaluate them by changing it to a distance. This is because a distance is usually used as a basis for classification, regression and other means of analyzing data.

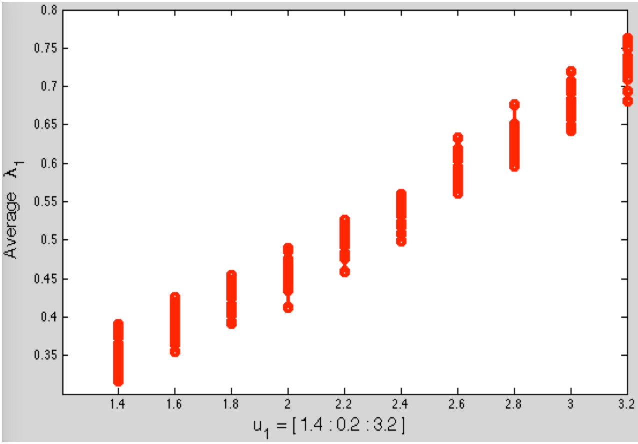

Figure 1. When changing u_1=[1.4: 0.2: 3.2], we will get ten conditions with each condition carried out for 20 times. We can see a subtle difference for each condition by using the average firing rate of multichannel spike trains.

The norm distance is a commonly used distance obtained from a kernel [5]. The norm distance between two spike trains is defined as follows.

$$d(t,s) = \sqrt{k(t,t) - 2k(t,s) + k(s,s)} \quad (7)$$

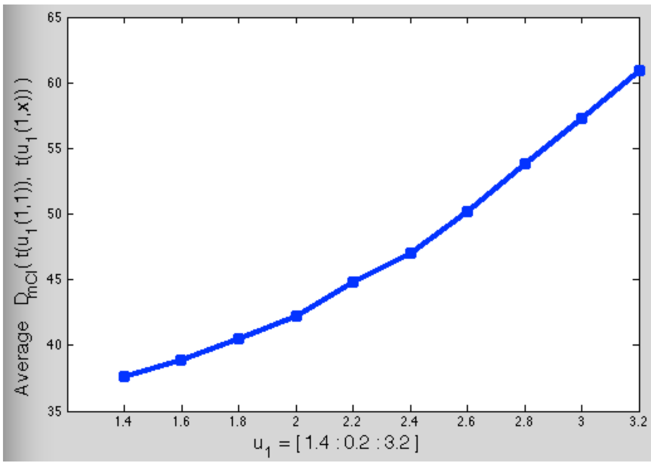

Figure 2. We calculate the norm distance for ten times, one is for the base condition $D_{mCI}(t_{u1},t_{u1})$, and others are between the base condition and each other condition $D_{mCI}(t_{u1},t_{ux})$, x = 2,3, … ,10.

As this experiment has proposed for ten conditions. In order to see the obvious difference, we set u_1 = 1.4, t=(t^1, t^2, t^3, … , t^{20}), as the base condition to compare to other conditions, s=(s^1, s^2, s^3, … , s^{20}).

From Figure 2, we can see a result that the distance of $D_{t,s}$

9 times is larger than the distance of $D_{t,t}$. This means that with the same u_1, the distance between spike trains (t^i, t^j) is small and with the different u_1, the distance between spike trains (t^i, t^j) is much smaller than the distance between spike trains (t^i, s^j).

5.3. Comparison with other parameters

Since we get the result by changing the parameter of u_1, it's necessary to prove this conclusion in the other way. So we do the same experiment with changing the parameter of J_{12} and a_1.

In the evaluation with a_1 =[-0.6: 0.05: -0.15], we used their original parameter settings which obtained u_1 =1.4. The result is showed in Figure 3 and Figure 4. AS there is a big change with the average firing rate, the difference of the norm distance is large.

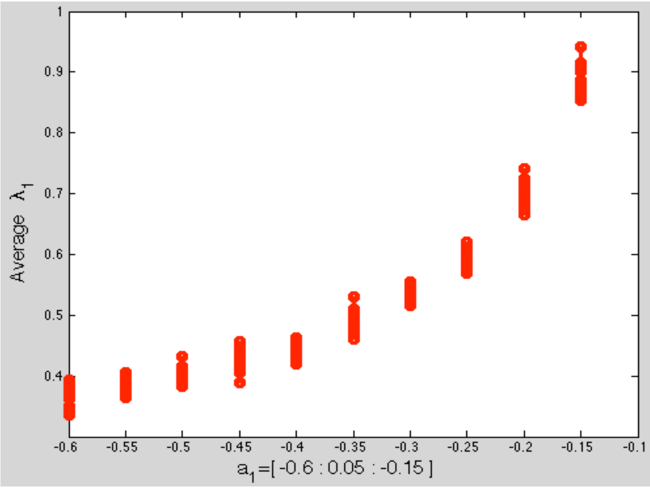

Figure 3. When changing a_1=[-0.6: 0.05: -0.15], the average firing rate of multichannel spike trains.

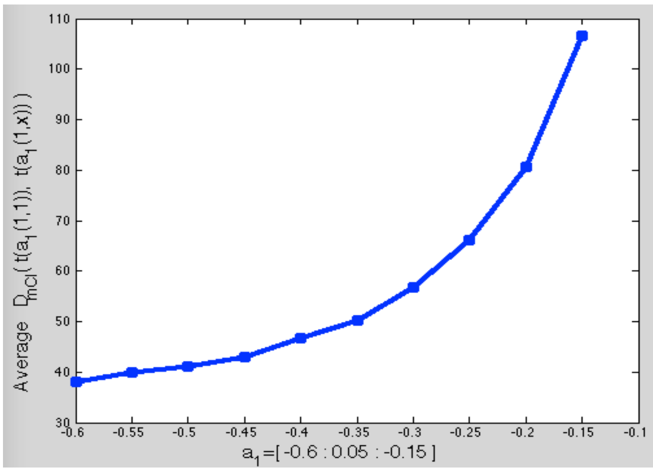

Figure 4. We consider a_1 = -0.6 as the base condition and calculate the norm distance for ten times.

For the condition of J_{12} =[-0.5: 0.05: -0.05], we also can the same result from Figure 5 and Figure 6.

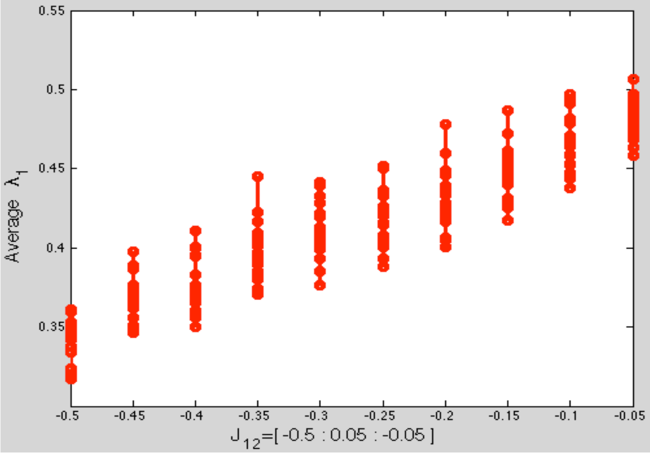

Figure 5. When changing J_{12}=[-0.5: 0.05: -0.05], the average firing rate of multichannel spike trains.

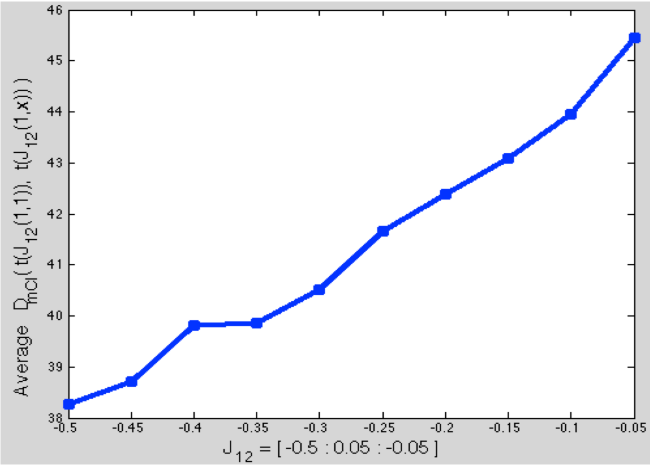

Figure 6. We consider J_{12} = -0.5 as the base condition and calculate the norm distance for ten times.

6. Conclusion

In order to analyze multichannel spike trains, we proposed a new kernel that extends memoryless cross intensity kernel. We evaluated it using simulated spike trains generated from the coupled escape rate model. The result showed that this method is effective at measuring multichannel spike trains.

In future work, we plan to extend this linear functional kernel to nonlinear functional kernel to evaluate their performance. We also plan to use other simulation models. They will make simulated data that more similar to the real spike trains.

Acknowledgements

This work was supported in part by JSPS KAKENHI Grant Numbers 21700121, 25280110, and 25540159.

7. References

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