

Inference of cooperations on the network based on the DFA by using a fixed-point observation

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Abstract By development of the Internet and spread of Internet devices among people, several activity and communications in social networks are done through the Internet. Many of activities via the Internet devices are observed at internet routers as well as access points, and their activities are recorded in the databases. In this talk, we investigate events at the fixed-point observation such as the access logs at the Internet routers or firewalls, and study the method to estimate the utilization situation of the Internet servers. From log data of the end-to-end connection and its service number, for example, we can know the direct utilization. However, it should be impossible to obtain links such that a pair of the events are caused by the same origin, because data acquisition is limited to the event passing through the observation point. In order to obtain such information, i.e., the correlation between events in the log database, we introduce time-series data analysis method, so called, the detrended fluctuation covariance analysis. The signal passing through an observation point are decomposed into signals of several channels, and activities of the decomposed channels are investigated by using the correlation function of the time-series data. In this talk, we examine the method applying to the log-data of the internet connections passing through firewall, and investigate the dynamical property of the usage of the servers and cooperation property between them.

Key words datamining, time series data, statistical reasoning

1. Introduction

Network structure and its dynamical phenomena such as Internet traffic are of significant interest. The Internet Protocol (IP) network is a self-organized network with no central control system, in contrast with ordinary communication networks such as the telephone network. The IP network can be expanded self-directively using routers, and communication-signal flows are controlled by the self-organized adjustment rule. The data are divided into packets, which are transferred by hopping between routers. Therefore, the waiting-line or exclusive-control mechanism may lead to a power-law fluctuation in IP traffic. In fact, such a power-law fluctuation in IP traffic is observed [1] [2] [3] [4] [5] just as for real traffic flow [6] [7] [8] [9] [11] [10].

On the other hand, the social-network activity on the Internet affects the dynamics of the Internet. By the recent spread of the mobile devices among people, many of activities and communications in social networks are done through the Internet. Many of activity via the Internet devices are observed at internet routes as well as access points. The influence of network structure such as free network and the dynamics of the network such as power-law fluctuation in

traffic flow can be observed in Internet traffic. For example, the mechanism of power-law correlations in case of sending E-mails are investigated in Refs. [12] [13] [14] [15] [16] [17] [18]. Possible origins of these power-law correlations include the structure of the Internet, internal mechanisms, and demands for the E-mail server.

It should be remarked that the dynamical behavior of the Internet traffic is determined not only by the direct use of the devices but also by the cooperation activity of the Internet servers. The Internet services work out on the communications between the Internet servers. The domain name service (DNS), for example, manages the FQDNs (full qualified domain name) and IP addresses for a local site, and answers the query to resolve the address of an Internet server. The DNS also send query to an external DNS to resolve the address at another site. Therefore, the Internet traffic is caused by the mixture of the communications.

In order to study the dynamical behavior of the Internet traffic such as power-law fluctuation and correlation profile between the Internet servers, we introduce the detrended fluctuation analysis (DFA) [19] [20]. The DFA is widely used to detect long range self-correlation in time-series data such as the stock price movement and traffic flow. Here, we ex-

tend the DFA so as to investigate the mixed time-series data, which we call detrended fluctuation covariance analysis (DFCA). In this talk, we focus on the cooperative behavior of the Internet dynamics. Here, we extend the DFA enable to investigate the mixed time-series data. We decompose the mixed signal of the Internet traffic into the selected channels associated to the Internet services such as E-mails, web access, and DNS. By applying the DFCA to the decomposed channels, we examine the independency or correlation between the Internet services. We further study the level of cooperation between DNS servers using the time-series data.

This paper is organized as follows. In Section 2., we explain the setup of our analysis and data processing for time-serise analysis from a database. In Section 3., we describe the method DFCA. In Section 4., we present analysis in detail. We investigate the power-law correlations, and then measure correlation function to investigate cooperation property between the Internet servers. In Section 5., we give summary and discussion. Finally, we give conclusion.

2. Time-series data processing

We investigate correlation and cooperation of categorized events in the Internet traffic by using the fixed-point observation at a firewall (FW). The FW divides into three domains, i.e., the Internet (WAN), the intermediate zone (DMZ), and local area network (internal-LAN). The internal-LAN is farther separated into several virtual LANs (subnets) by using the firewall. Figure 1 shows the sketch of network topology which we take out data from The FW monitors and filters point-to-point IP communications across these segments, and it records the logs of all requested TCP/IP connections. The log records include data such as connecting time, duration, source and destination IP addresses, the port number of both endpoints, transfer data size, the flag whether the connection is permitted or not, and so on. Therefore, using the log data we can reconstruct Internet connections via the FW in the form of time-series data.

In this paper, we use log data from FW at KEK. In the log file there are many fields for connection, we only use a part of fields.^(注1) We drop the IP address after classification of segments. Table 1 summarizes the properties of the data used. The LAN denotes the segments including the DMZ segment just as internal subnets (see Fig. 1). We use the data extracted from logs for a 36 weeks period from 4 May 2009 to 10 January 2010, and we eliminate connections that are blocked by the FW and also those from network monitoring systems. We classify the data into two data types:

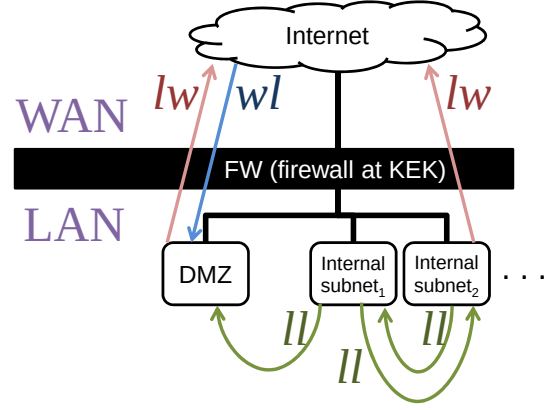


Figure 1 Sketch of IP traffic for our analysis. The network is divided into the segments WAN, DMZ, and LAN. The FW controls whether the IP traffic passes through the network segment. The blue arrow labeled wl represents the connection from WAN to DMZ. The red arrows labeled lw represent the connections to WAN from LAN (internal subnets or DMZ). The green arrows labeled ll represent the connections between segments in LAN. Connections from WAN to internal subnets and from DMZ to internal subnets are denied by the FW.

one is *conn* (connection) which is the number of permitted connections, and the other is *flow* (data flow) which is the amount of data transferred. These two types of data are further classified in terms of Internet services (protocols) and the direction of TCP/IP connections. In terms of the direction of TCP/IP connections, the data are divided into three groups labeled by wl , lw , and ll , as shown in Fig. 1.

Time frame		from 4 May 2009 to 10 Jan 2010 through 36 weeks
Data type	<i>conn</i>	the number of connections
	<i>flow</i>	the amount of data transfer
Service (protocol)	<i>mail</i>	SMTP (25)
	<i>web</i>	HTTP (80), HTTPS (443)
	<i>dns</i>	DNS (53)
	<i>others</i>	others than above
Network zone	WAN	Internet
	LAN	DMZ, internal subnets (VLANs)
Transfer class	<i>wl</i>	from WAN to DMZ (LAN)
	<i>lw</i>	from LAN to WAN
	<i>ll</i>	among DMZ and internal subnets

Table 1 Summary of data used.

The label wl indicates the connection from WAN to DMZ (LAN), lw indicates the connection from LAN to WAN, and ll indicates the connection between segment pairs in the LAN segments. In terms of services (protocols), we divide the data into selected services such as *web*: the connection at port 80 (HTTP) or 443 (HTTPS), *mail*: the connection at port 25 (SMTP), *dns*: the connection at port 53 (DNS), and *others*:

(注1) : We get permission to access and use the classified information after masking the IP addresses.

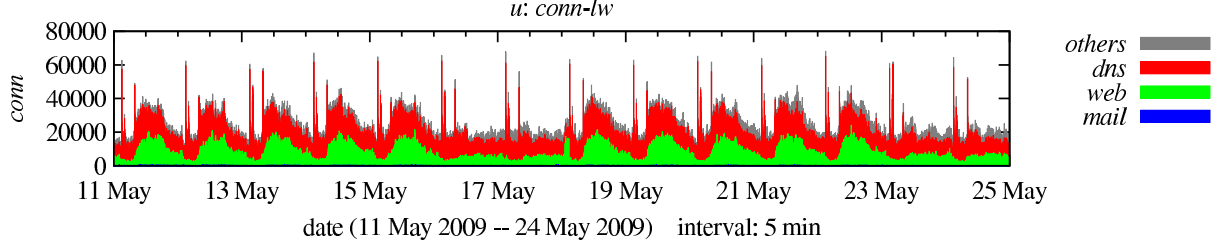


Figure 2 Time-series data $\bar{u}^{(k)}(n)$ ($k \in \{ \text{mail}, \text{web}, \text{dns}, \text{others} \}$) for the *conn-lw* channels. The period is from 11 May to 24 May 2009. The number of connections is shown after every 5 min. interval. The connections are superimposed on each other from bottom to top: *mail*(blue), *web*(green), *dns*(red), and *others*(gray).

connections at other ports. Therefore, we have 24 channels: $\{\text{conn}, \text{flow}\} \otimes \{\text{web}, \text{mail}, \text{dns}, \text{others}\} \otimes \{\text{wl}, \text{lw}, \text{ll}\}$.

In what follows, we use the notation “(data type)-(service)-(transfer class)” or a portion thereof to represent the categorized data, so-called channel, e.g., *conn-mail-lw*, *conn-mail*, etc. For example, sending an E-mail from LAN to WAN is recorded in the channel *conn-mail-lw*, and its size is recorded in *flow-mail-lw*, while receiving an e-mail from WAN is recorded in *conn-mail-wl* and its size is recorded in *flow-mail-wl*. In the same way, browsing through KEK from the Internet is recorded *page by page* in *conn-web-wl* and its size is recorded in *flow-web-wl*.

In this paper, we use RDBMS, Oracle12c, to analyze the time-series data. The RDBMS has fast and strong condition-matching functionality for huge size of data, and various useful functions for data mining.

First, from the FW logs we extract record such as connecting time, a pair of IP addresses of the point-to-point TCP/IP connection, access-permission flag, connection duration, and data size (see Table 1), and store them into a database as the raw data. We have about 10 million of records per day, and about 3 billions of the amount of records in 36 weeks within permitted connections.

We further invite various technique of the RDBMS to gain reasonable processing performance such as the table-separation technique and the star-schema structure. For example, the FW logs are stored into the daily tables by using the table-separation technique, without loss of the seamless access to the tables. With the table-separation, each of the daily tables has relatively a small number of records in 10 million instead of a whole number in 3 billions. The daily tables have no relations each other and we can issue queries in parallel. Furthermore, with the star-schema structure, each table is treated as a fact table and does not have text data directly such as “http”, “permit”, “tcp”, and so on. The raw text data are stored into dimension tables that compresses a large-size dimension tables considerably. Consequently, the computation speed improves considerably and we can calcu-

late various channels in Table 1. The aggregation functions are useful to calculate the histogram of the number of connections in one minute or five minutes, and to classify the transfer class, $\{\text{wl}, \text{lw}, \text{ll}\}$ from point-to-point connections.

Fig. 2 shows a part of $u^{(k)}(t)$ for *conn* for the two-week period from 11 May to 24 May 2009, where each $u^{(k)}(t)$ of *mail*, *web*, *dns*, and *others* is superimposed on the others for each five minutes interval. In Fig. 2, we find daily frequencies and weekly frequencies. These periodic trends in $u^{(k)}(t)$ must be eliminated separately.

3. Method

The detrended fluctuation analysis (DFA) is originally presented to analyze the non-static data by applying the root mean square analysis of a random walk [19], [20].

Let $\hat{u}(t)$ a mixed time-series signal in a period T and $\hat{u}^{(k)}(t)$ a decomposed signal classified by the K categories. The time-series dataset $\{u_n^{(k)}\}$ is obtained as the time-sliced signal with a period Δt :

$$u_n = \sum_{k=1}^K u_n^{(k)}, \quad u_n^{(k)} = \int_{t_{n-1}}^{t_n} \hat{u}^{(k)}(s) ds, \quad (1)$$

where we define $t_n = t_{n-1} + \Delta t$ and $n = 0, 1, \dots, N_T = T/\Delta t$.

To each classified dataset, we apply conventional DFA. We define y -function for each signal, $y_n^{(k)}$:

$$\begin{aligned} y_n^{(k)} &= \int_{t_0}^{t_n} \left(\hat{u}^{(k)}(s) - \langle u^{(k)} \rangle \right) ds \\ &= \sum_{t=1}^{N_t} \left(u_t^{(k)} - \langle u^{(k)} \rangle \right), \end{aligned} \quad (2)$$

$$\langle u^{(k)} \rangle = \frac{1}{T} \int_{t_0}^{t_n} \hat{u}^{(k)}(s) ds = \frac{1}{N_t} \sum_{t=1}^{N_t} u_t^{(k)}. \quad (3)$$

where we define an average value $\langle u^{(k)} \rangle$. Then, we divide the period T into M -divided intervals with period $T_M = T/M$: $I^{(M)} := [t_m^{(M)}, t_{m+1}^{(M)}]$ with $t_m^{(M)} = t_0 + mT_M$, $m = 1, \dots, M$. For the m -th interval we define a trend function using linear function, $\tilde{y}_{m:M}^{(k)}(t)$, which is determined by a least-square fit of the data in the m -th interval. Therefore, we obtain the

detrended fluctuation function $\Delta y_{m;M}^{(k)}(t)$ defined by

$$\Delta y_{m;M}^{(k)}(t) := y^{(k)}(t) - \tilde{y}_{m;M}^{(k)}(t). \quad (4)$$

Therefore, we obtain correlation function between k and k' channel.

$$R^{(k,k')}(T_M) := \sum_{m=1}^M \sum_{t_n \in I(M)} \Delta y_{m;M}^{(k)}(t_n) \Delta y_{m;M}^{(k')}(t_n) \quad (5)$$

By using variance mode, we measure the power coefficient $\alpha(k)$ in the DFA,

$$F^{(k)}(T_M) := R^{(k,k)}(T_M) \sim (T_M)^{\alpha(k)}. \quad (6)$$

The power coefficient $\alpha(k)$ shows the profile of the dynamics in the channel k such as the self-organized $\alpha(k) \simeq 1$ or random. By using the covariance mode, we obtain the information of independent or cooperative. We introduce the normalized index, i.e., covariant coefficient :

$$\rho^{(k,k')}(T_M) := \frac{R^{(k,k')}(T_M)}{\sqrt{F^{(k)}(T_M)} \sqrt{F^{(k')}(T_M)}} \in [-1, 1], \quad (7)$$

4. Analysis

In this section, we study the correlation between selected channels. We have decomposed channels, $\{conn, flow\} \otimes \{web, mail, dns, others\} \otimes \{wl, lw, ll\}$, we have possible 300 correlation functions, i.e., 24 F -functions and 276 R -functions. However, most of them should vanish, because the origins of the fluctuations can be independent. In this paper, we focus the cooperation of the Internet servers.

Note that the RDBMS is useful in the DFA analysis. SQL enables us complex calculation with complex condition matching by using aggregation with multiple indexes, window functions, and conditional expressions, we can further calculate y -function, Eq.(2), which is integral of $u(t)$ with subtraction of weekly average.

4.1 Variance and self-organized process

We investigate the categorized data by using DFA. Fig.3 plots the y -functions for the *conn-web-lw* channel. The green line represents y -function for the original data. Since we have a daily and weekly periodic-profile, we extract this periodic profile by using a frequency trend-function. We define the frequency trend-function with T_Q frequency as the average of $u_n^{(k)}$ function of the interval T_Q :

$$\tilde{u}_Q^{(k)}(t_n) = \frac{1}{N_Q} \sum_{m=0}^{N_Q-1} u_{n+m(T_Q/\Delta t)}^{(k)}, \quad (8)$$

$$(n = 0, \dots, T_Q/\Delta t - 1)$$

where T_Q denotes the period, i.e., day or week, Δt is the sampling time interval, and $N_Q = T/T_Q$ is the cycle length.

	WEB	SMTP	Others	DNS
WEB	-	×	×	○
SMTP	×	-	×	○
Others	×	×	-	△
DNS	△	△	△	-

Table 2 Covariance coefficients for connections. Upper triangle represents the coefficients for LW channels, while lower triangle for WL channels. A symbol ○ stands for the clear correlation relation, and △ stands for the weak correlation.

Thus, the detrended time series data $\bar{u}_Q(t)$ is given by

$$u_{Q,n}^{(k)} = u_n^{(k)} - \tilde{u}_Q^{(k)}(t_n \bmod T_Q). \quad (9)$$

The blue line in Fig.3 shows y -function of detrended weekly frequency. It shows that both the daily and weekly trends are eliminated from the original data in the detrended y -function, i.e., the bumpy curve with the daily and weekly periods (green curve) is smoothed.

Fig. 4 shows the $F^{(k)}(T_M)$ function, Eq.(6), for the *web-wl*, *email-wl*, and *dns-lw* channels. For each channel, the F -functions are plotted for both the weekly detrended and the non detrended data: the aqua and green plots represent the non-detrended F -functions for *flow* and *conn*, and the red and blue plots represent the weekly-detrended F -functions. The curving disappears from both the weekly-detrended plots shown in red and blue in Fig.4. We then fit the fitting with a linear function over the whole range of the weekly-detrended data. Thus, we find power-law fluctuation in *conn-web-lw* and *flow-web-lw* channel.

4.2 Covariance and cooperation

First, we analyze the *conn* channels, $\{conn\} \otimes \{web, mail, dns\} \otimes \{wl, lw, ll\}$, which exhibit power-law correlations in the DFA. Fig. 5 shows the correlation coefficients, Eq.(7), for the three directions *wl*, *lw*, and *ll*. For long time scale, we obtain scattered data in the coefficients, which are caused by the large error in the correlations. In this study, therefore, we focus on the data of $T_M < 2$ days ($\simeq 3000$ min).

We obtain vanishing coefficient $\rho(web, mail) \simeq 0$; therefore, the activities of web browsing and of sending e-mail are independent. However, we find correlation coefficients, $\rho(dns, mail)$ and $\rho(dns, web)$, are nonvanishing. These results suggest cooperation between servers and clients in the Internet which is prescribed in the Internet protocols. We summarize the result of correlation in Table 2.

Next, we examine the correlation coefficients between the *dns* channels to understand in detail the level of cooperation between DNS servers distributed in the Internet in detail. Fig.6 shows the correlation coefficients for the *dns* channels of three-directions, i.e., $\rho(dns-wl, dns-lw)$, $\rho(dns-wl, dns-ll)$, and $\rho(dns-lw, dns-ll)$. We obtain vanishing or very small

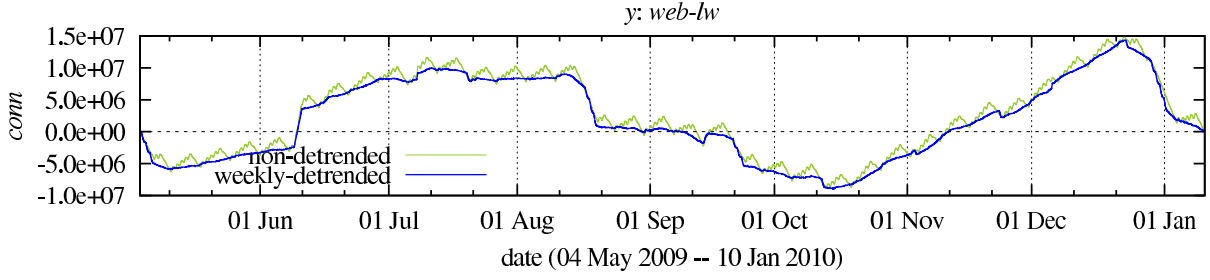


Figure 3 y -function, Eq.(2), for *web* service in *conn* data from LAN to WAN (*conn-web-lw*). The green line shows *conn-web-lw* with non-detrended, and the blue line shows the same thing but weekly-detrended.

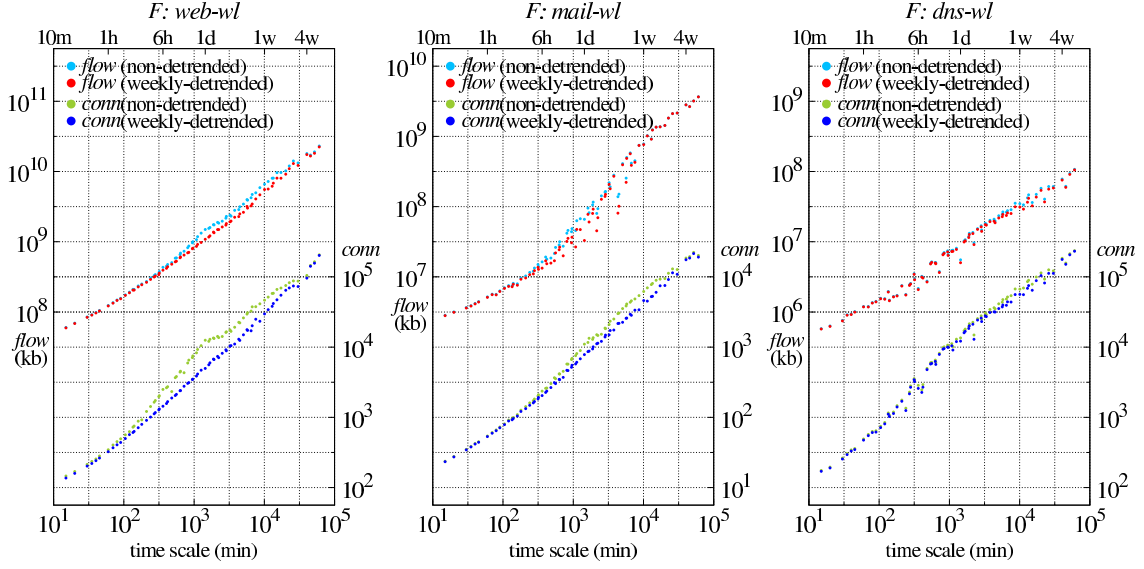


Figure 4 Combination plots of F -function for the *web* channel.

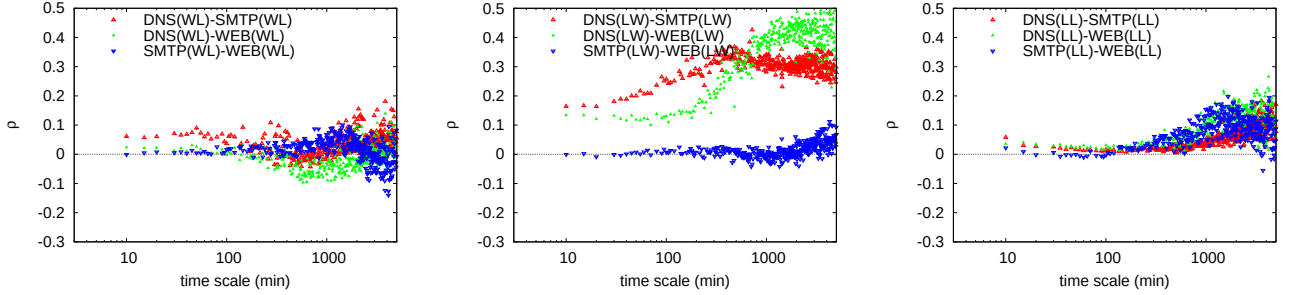


Figure 5 Correlation coefficients $\rho(T_M)$ between *web*, *mail*, and *dns* channels classified by direction *wl*, *lw*, and *ll*. From upper to lower panels are shown the coefficients of the *wl*-direction, *lw*-direction, and *ll*-direction, respectively. For each direction, the red points represent $\rho(dns, mail)$, $\rho(dns, web)$, and $\rho(mail, web)$, respectively.

correlation coefficients $\rho(dns-wl, dns-ll)$, which is attributed to the DNS servers holding only the data of the local domain. However, nonvanishing $\rho(dns-wl, dns-lw)$ and $\rho(dns-lw, dns-ll)$ indicate that the DNS servers cooperate. The nonvanishing coefficient $\rho(dns-lw, dns-ll)$, for example, is caused by web browsing, i.e., the DNS queries for the *dns-lw*

and *dns-ll* directions occur at the same time because of the cooperation between DNS servers as discussed in the previous paragraph. While, the nonvanishing coefficient $\rho(dns-wl, dns-lw)$ can be caused, for example, by receiving e-mails from the Internet. The DNS query *dns-wl* is caused by transferring e-mail from WAN to LAN, and the mail server may

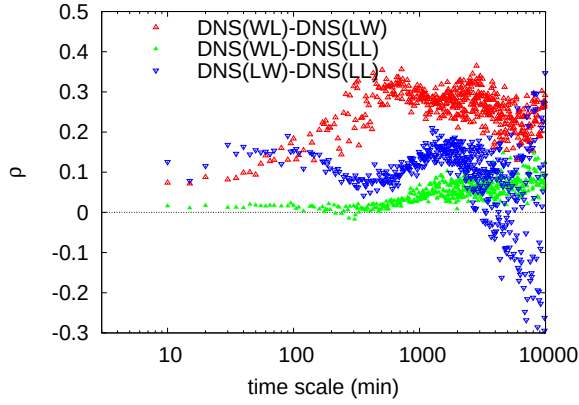


Figure 6 Correlation coefficients between dns channels. Red, green, and blue points represent $\rho(dns - wl, dns - lw)$, $\rho(dns - wl, dns - ll)$, and $\rho(dns - lw, dns - ll)$, respectively.

send the invert request to resolve a FQDN from the IP address of which transfers the e-mail.

Next, we investigate correlation coefficients between the *conn* and *flow* channels. This is a test for the randomness of data size for each connection. Fig.7 shows the covariance coefficients between the *conn* and *flow* data for the same channel id, (i.e., the pair of service-id and direction-id such as *dns-ll*). The coefficients for the *dns* service reveal a strong correlation between the *conn* and *flow* channels, which suggests the data size for each query is the almost same. The coefficients for the *mail* service reveal a weak correlation, which suggests that e-mail size is random. For the *web* service, we find a strong correlation between the *conn* and *flow* data type compared with that for the *mail* service. This suggests that the web page size is mostly random, but over the long run the average web page size is almost the same.

5. Summary and Discussion

We investigated IP traffic via Internet communication using firewall log. Internet traffic is the effect of network dynamics and associated activities of the social network with Internet services. Toward this end, we extended the DFA into the DFCA which allows us to investigate the mixed time-series signals. We classified IP traffic in terms of the type of IP connection, and decomposed the signal into the channel signal, (i.e., the channel is classified according to data type, service type, and direction of connection).

Using the covariance analysis, we examined the level of independence of the sources of the fluctuation and of the cooperation in the Internet traffic. We find the vanishing covariance coefficient $\rho(mail, web)$ between *web* and *mail* chan-

nels, i.e., we consider that the activities of web browsing and mail sending are independent. In contrast, we find nonvanishing coefficients between *dns* and *mail*, and between *dns* and *web*. These results reflect the dynamics of Internet traffic, i.e., the cooperation between the mail and DNS servers, and between the web and DNS servers. We further examined the level of the cooperation between DNS servers distributed over the Internet using covariance coefficients. We find the nonvanishing covariance coefficients between the *ll* and *lw* directions and between the *lw* and *wl* directions. It should be remarked that these cooperations between Internet servers are found without additional information such as Internet server protocols but using time-series data only.

These results are understood as the fact of cooperation between servers and clients in the Internet which is prescribed in the Internet protocols. For web browsing, for example, an address by FQDN^(註2) is used in the URL (see Fig.8). To access to the web server, the IP address of the web server must be resolved from the FQDN. Then, the web browser usually queries the IP address for the DNS server of neighbors. Thus, web browsing causes a query for the DNS server. Whether the query is recorded in the logging file of the FW depends on the status of the DNS server. Because DNS servers are distributed databases in which data is hierarchically organized, only the IP addresses of the local domain are held. If the IP address is stored in the local database or cached in memory, the DNS server can quickly answer the query. However, if a DNS server receives the request for an unknown address, it asks another DNS server to resolve the address. This causes another query for the DNS server, and this request can be recorded in case the query steps over the FW.

The covariance analysis thus detects and classifies the dynamical property of the Internet such as the cooperation between servers and the effect of the social activity via the Internet. Some of the results can be derived from the protocols and the mechanism of the Internet services. However, these results are obtained without using any knowledge of Internet services in analyzing the time series data.

6. Conclusion

From these results in the DFCA, we arrive at the following concluding remarks. Using the covariance coefficients for the detrended fluctuation signals, we can examine the cooperation profile of classified channels. The vanishing correlation-coefficient shows independence of the pair of channels, while the nonvanishing correlation-coefficient shows the relationship between the channel pairs. Within this analysis, we

(註2) : FQDN stands for full qualified domain name, which in layman's term is called Internet address.

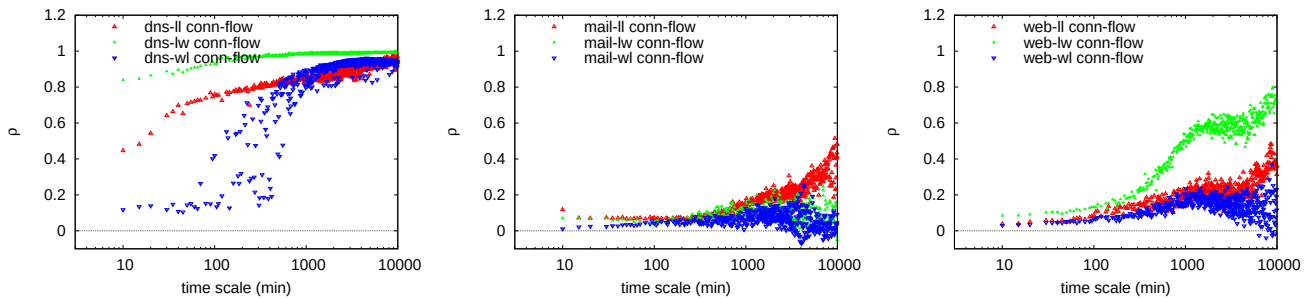


Figure 7 Correlation coefficient between *flow* and *conn* channels. The left, middle, and right panels show the coefficients for *dns*, *mail*, and *web*, respectively. For each panel, the red, green, and blue points represent the *ll*-direction, *lw*-direction, and *wl*-direction, respectively.

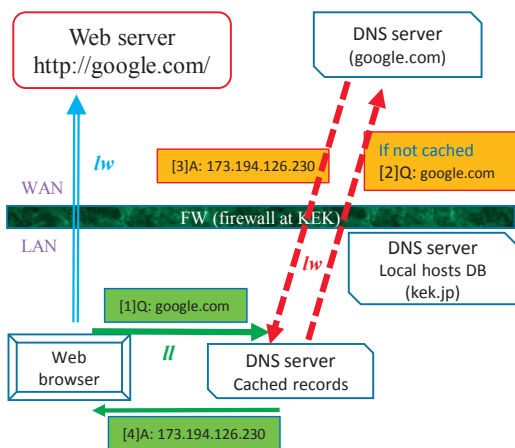


Figure 8 Sketch of the cooperation between DNS servers.

can correctly detect the cooperation between Internet servers even when using the restricted informations obtained by fixed-point observation.

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References

- [1] V. Paxson and S. Floyd, IEEE/ACM Trans. Networking 3, 226 (1995)
- [2] I. Csabai, J. Phys. A 27, L417 (1994)
- [3] M.Takayasu, H.Takayasu and T. Sato, Physica A233 (1996) 824-834
- [4] Masao Masugi, IEICE technical report. Information networks Vol.104, No.121 (20040611) 7-12
- [5] Shin-ichi Tadaki, J.Phys. Soc. Jpn. 76 044001(2007)
- [6] T.Musha and H.Higuchi, Jpn. J. Appl. Phys. 15 (1976) 1271-1275
- [7] M.Takayasu, A. Tretyakov and K. Fukuda and H. Takayasu, in Traffic and Granular Flow'97 rd. D.E. Wolf (Springer, Berlin, 1998) p.57
- [8] K.Fukuda, H.Takayasu: Fractals 1 (1993) 916
- [9] S. Yukawa and M.Kikuchi: J.Phys Soc. Jpn. 65 (1996) 916
- [10] S.Tadaki, M.Kikuchi, Y.Sugiyama and S. Yukawa: J. Phys. Soc. 68 (1999) 3110
- [11] S.Tadaki, M.Kikuchi, A. Nakayama, K. Nishinari, A. Shibata, Y. Sugiyama and S. Yukawa: J. Phys. Soc. 75 (2006) 0344002
- [12] J.-P. Eckmann, E. Moses and D. Sergi, Proc. Natl. Acad. Sci. USA 101, 14333 (2004)
- [13] A.-L. Barabási, Nature 435, 207 (2005)
- [14] K.-I. Goh, A.-L. Barabási and Europhys. Lett. 81, 48002 (2008)
- [15] R.D. Malmgren, D.B. Stouffer, and A.E. Motter, L.A.N. Amaral, Proc. Natl. Acad. Sci. USA 105, 18153 (2008)
- [16] C. Anteneodo, R.D. Malmgren, and D.R. Chialvo, Eur. Phys. J. B 75, 389 (2010)
- [17] M. Karsai, K. Kaski, A.-L. Barabási, and J. Kertész, Sci. Rep. 2, 397 (2012)
- [18] Y. Matsubara, Y. Hieida, and S. Tadaki, Eur. Phys. J. B, 86 9 (2013) 371.
- [19] C.-K. Peng, S.V. Buldyrev, S. Havlin, M. Simons, H.E. Stanley and A.L. Goldberger, Phys. Rev. E49,1685-1689 (1994)
- [20] C.-K. Peng, S. Havlin, H.E. Stanley and A.L. Goldberger AL, Chaos, 5(1), (1995) 82-87
- [21] K. Hu, P.Ch. Ivanov, Z. Chen, P. Carpena, and H.E. Stanley, Phys. Rev. E 64, 011114 (2001)
- [22] S. Tadaki, Comp. Phys. Commun. 182, 237 (2011)